But these are the equations of motion of a particle, of mass M, placed at the centre of inertia of the body and acted on by forces parallel to, and equal to, the external forces acting on the different particles of the body.

Hence the centre of inertia of a body moves as if all the mass of the body were collected at it, and as if all the external forces were acting it in the directions parallel to those in which they act.

Let x', y', z' be the co-ordinates relative to the centre of inertia G, of a particle of the body whose co-ordinates referred to the original axes ~~where~~ were x, y, z.

Then $x = \bar{x} + x'$, $y = \bar{y} + y'$ and $z = \bar{z} + z'$

throught the motion.

$$\therefore \frac{d^2x}{dt^2} = \frac{d^2\bar{x}}{dt^2} + \frac{d^2x'}{dt^2}$$

$$\frac{d^2y}{dt^2} = \frac{d^2\bar{y}}{dt^2} + \frac{d^2y'}{dt^2}$$

$$\frac{d^2z}{dt^2} = \frac{d^2\bar{z}}{dt^2} + \frac{d^2z'}{dt^2}$$

$$\therefore y\frac{d^2z}{dt^2} - z\frac{d^2y}{dt^2}$$

$$= (\bar{y}+y')\left(\frac{d^2\bar{z}}{dt^2} + \frac{d^2z'}{dt^2}\right) - (\bar{z}+z')\left(\frac{d^2\bar{y}}{dt^2} + \frac{d^2y'}{dt^2}\right).$$

Hence the eqn (4) of the last article gives

$$\Sigma m\left(\bar{y}\frac{d^2\bar{z}}{dt^2} - \bar{z}\frac{d^2\bar{y}}{dt^2}\right) + \Sigma m\left(y'\frac{d^2z'}{dt^2} - z'\frac{d^2y'}{dt^2}\right)$$

$$+ \Sigma m\left[\bar{y}\frac{d^2z'}{dt^2} + y'\frac{d^2\bar{z}}{dt^2} - \bar{z}\frac{d^2y'}{dt^2} - z'\frac{d^2\bar{y}}{dt^2}\right]$$

$$= \Sigma\left[(\bar{y}+y')Z - (\bar{z}+z')Y\right] \quad \text{—— (4)}$$

Now $\dfrac{\Sigma my'}{\Sigma m}$ = the y co-ordinate of the centre of inertia referred to G as origin $= 0$

and therefore $\Sigma my' = 0$

and $\Sigma m\dfrac{d^2y'}{dt^2} = 0$

so $\Sigma mz' = 0$ and $\Sigma m\dfrac{d^2z'}{dt^2} = 0$

Hence (4) gives

$$M\left[\bar{y}\frac{d^2\bar{z}}{dt^2} - \bar{z}\frac{d^2\bar{y}}{dt^2}\right] + \Sigma m\left(y'\frac{d^2z'}{dt^2} - z'\frac{d^2y'}{dt^2}\right)$$

$$= \Sigma[\dot{y}Z - \dot{z}Y + y'Z - z'Y] \quad \ldots\ldots (5).$$

But equations (2) and (3) give

$$M\left[\bar{y}\frac{d^2\bar{z}}{dt^2} - \bar{z}\frac{d^2\bar{y}}{dt^2}\right] = \Sigma[\bar{y}Z - \bar{z}Y] \quad \ldots\ldots (6)$$

But this equation is of the same form as equation (4) of the last article, and is thus the same equation as we should have obtained if we had regarded the centre of inertia as a fixed point.

Hence the motion of a body about its centre of inertia is the same as it would be if the centre of inertia were fixed and the same forces acted on the body.

The two resultants proved in the previous article shew us that the motion of rotation.

By the first result we see that the motion of the centre of inertia is to be found by the methods of Dynamics of a Particle.

By the second result we see that the motion of rotation is reduced to finding that of a body about a fixed point.

As a simple example, consider the case of a uniform stick thrown given direction and at the same time it is rotating with given angular velocity about its centre. By the first result the motion of the centre of inertia is the same as if there were applied at it all the external forces acting on the body in directions parallel to that in which they act. In this case these external forces are the weights of the various elements of the body; when applied at the centre of inertia they are equivalent of the total weight of the body. Hence the centre of the stick moves as if it weres a particle of Mass M acted on by a vertical forces Mg, i.e. it moves just as a particle would under gravity if it were projected with the same velocity as the centre of the stick. Hence the path of the centre of the stick would be a parabolla.

In a subsequent chapter it will be seen that the angular velocity of the stick will remain unaltered. Hence the centre of the stick will describe a parabola and the stick revolve uniformely about it.

As another example consider a shell which is in motion in the air and suppose that it bursts into fragments. The internal forces exerted by the explosion balance one another, and do not exert any influence on the motion of the centre of inertia of the shell. The centre of inertia therefore continues to describe the same parabola in which it was moving before the explosion. [The motion is supposed to be in vacuo and gravity to be constant.

Equation (1) of Art. 161 may be written in the form:—

$$\frac{d}{dt}\left[\Sigma m \frac{dx}{dt}\right] = \Sigma(X),$$

i.e. $\frac{d}{dt}$ [Total momentum parallel to the axis of x]

= sum of the impressed forces parallel to OX.

So for the other two axes,

Also (4) can be written:—

$$\frac{d}{dt}\left[\Sigma m\left(y\frac{dz}{dt} - z\frac{dy}{dt}\right)\right] = \Sigma(yZ - zY),$$

i.e. $\frac{d}{dt}$ [Total moment of momentum about the axis of x]

= Sum of moments of the impressed forces about OX.